Chapter 7 - Section B

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Exercises

Ex. 01

False. If there is a basis consisting of eigenvectors of T, Then $M(T)$ is diagonal. It follows $M(T)M(T)^* = M(T)^*M(T)$, Equivalently $TT^* = T^*T$, So T is self-adjoint.

Ex. 02

Assume $F = \mathbb{R}$.

Observation. $p(x) = x^2-5x+6 = (x-2)(x-3)$. $p(T) = T^2-5T+6I = (T-2I)(T-3I)$

The goal is $p(T) = 0$. It suffices to show $p(T)v = 0$ for any vector v.

By *Real Spectral Theorem* (p. 221), There is a basis of eigenvectors of T corresponding to eigenvalues $\lambda_1, .., \lambda_n$. By hypothesis we know $\lambda_i = 2$ or $\lambda_i = 3$.

Let v be an arbitrary vector v. Then $v = a_1v_1 + \cdots + a_nv_n$. Observe $p(T)(v) = p(T)(a_1v_1)+\cdots+p(T)(a_nv_n) = a_1p(\lambda_1)v_1+\cdots+a_np(\lambda_n)v_n$. But $p(\lambda_i) = 0$ so $p(T)v = 0$.