Ch.08, Sec.01 - Bartle & Sherbert. Real Analysis

Mostafa Touny

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Contents

Exercises

Ex. 11

For $f_n(x) = \frac{x}{x+n}$, and $f = 0$, Clearly $||f_n - f||_{[0,a]} = ||f_n||_{[0,a]} = \frac{a}{a+n}$. But $\lim_{n\to\infty} \frac{a}{a+n} = 0$. Hence by *lemma 8.1.8* (page 244), The uniform convergence on $[0, a]$ follows.

We follow *Lemma 8.1.5* (page 244). Consider subsequences $n_k = x_k = k$. Then $f_{n_k}(x_k) = \frac{k+k}{k} = \frac{1}{2}$. Therefore $|f_{n_k}(x_k) - f(x_k)| = |f_{n_k}(x_k)| = \frac{1}{2} = \epsilon_0$.

Ex. 18

We use *lemma 8.1.8* (page 244). Note $f_n(x) = xe^{-nx}$ and $f = 0$. Then $||f_n - f||_{[0,\infty)} = ||f_n||_{[0,\infty)} = 1/n$.

To see why, Observe $f_n'(x) = (e^{-nx})(1-nx)$, and setting $f_n'(x) = 0$ yields local max/min at $x = 0$ and $x = 1/n$. That justifies the supremum we aforementioned.

But $\lim_{n\to\infty} 1/n = 0$, Concluding uniform convergence.

Ex. 21

Observe $|(f_n(x)+g_n(x))-(f(x)+g(x))| \leq |f_n(x)-f(x)|+|g_n(x)-g(x)| < \epsilon/2+\epsilon/2 = \epsilon$, Following by the triangle inequality.