Mostafa Touny mtouny@msa.edu.eg
Salma Abdelhalim salmaemad@aucegypt.edu
Ali Nadeem nadim99@aucegypt.edu
Omar Shaalan shaalan@aucegypt.edu

Exercise 1

i)

Def. A graph is finite if V is finite, i.e its order is finite

Assumption. $|G| = |V|$, V finite

An edge is defined by 2 vertices. Since E contains all edges, and they're of the form $\{u,v \mid u,v \in V\}$, Therefore edges number is finite

True

ii)

Assumption. E finite $\rightarrow$ G finite

a graph G can have a finite set of edges E. However its cardinality can be infinite as we can have graphs that has unconnected vertices, and the vertices can be infinite

False

Exercise 2

$|G| = |V| \qquad \|G\| = |E|$

$V = \{v_0, v_1, \dots, v_n\}$

$E = \{e_1, e_2, \dots, e_{n-1}\}$

The least possible $\delta(G)$ that can exist is 0, and the maximum can be found in complete graphs where a vertex v_n is connected to $n-1$ vertices

$\therefore$ We can re-write this corollary $\delta(G) \leq 2\,|E|/|V| \leq \Delta(G)$ as $0 \leq 2\,|E|/|V| \leq n-1$

i.e

$$|E| / |V| \leq \frac{n-1}{2}$$

We can replace $|V|$ with the real cardinality which is the number of vertices, n

$$|E| \leq \frac{n-1}{2} / n$$

$$\leq \frac{n(n-1)}{2}$$

Exercise 3

Assumption. Any graph G has an even number of vertices of odd degree

We Know $\sum_{v \in V} \deg(v) = 2|E|$

Exercise 4

$N(u)$ is the set of all vertices adjoint to vertex u
Similarly $N([u,v])$ is the set of all vertices that are adjoint to the vertices u and v
To show $N(\{u,v\}) = N([u,v])$, We use double inclusion.

$(\rightarrow)$
$w \in N(\{u,v\})$. Then w is adjoint to either u or v. it follows it's adjoint to both u and v. So $w \in N([u,v])$

($\Leftarrow$)

let $w \in N([u,v])$. Then w is adjoint to both u and v.

Hence $w \in N(\{u,v\})$. Q.E.D