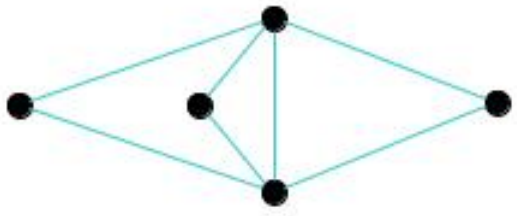
Graph Theory
Homework Series 02
In Groups of 2-4 Students

Keywords: Regular Graphs, Complements, Subgraphs, Walks.

Exercise 1 (Complements)**.** *Consider the graph G given below.*



(i) Determine the order, size and degree sequence of G ***and*** *its complement.*

(ii) For an arbitrary graph G of order n and size k and degree sequence $(k_1, \ldots, k_n)$, determine order, size and degree sequence of its complement.

2+4 points.

Exercise 2 (Regular Graphs)**.** *Prove or disprove the following claim:*
Let G be an 11-regular graph. Then G is of even order.

2 points.

Exercise 3 (Neighborhoods - Gouda's Lemma :))**.** *Prove that for any graph G and any subset $S \subseteq G$ we have*

$$N(S) = N[S] \text{ if and only if } \delta(\langle S \rangle) \geq 1.$$

5 points.

Exercise 4 (Walks and Cycles)**.** *Let G be a graph with $\delta(G) \geq k$.*

(i) Show that G contains a path of length at least k.

(ii) If $k \geq 2$, show that G has a cycle of length at least $k + 1$.

4+(2 Bonus) points.

Exercise 5 (Walks)**.** *Prove that any closed odd walk in a graph contains an odd cycle.*

3 points.