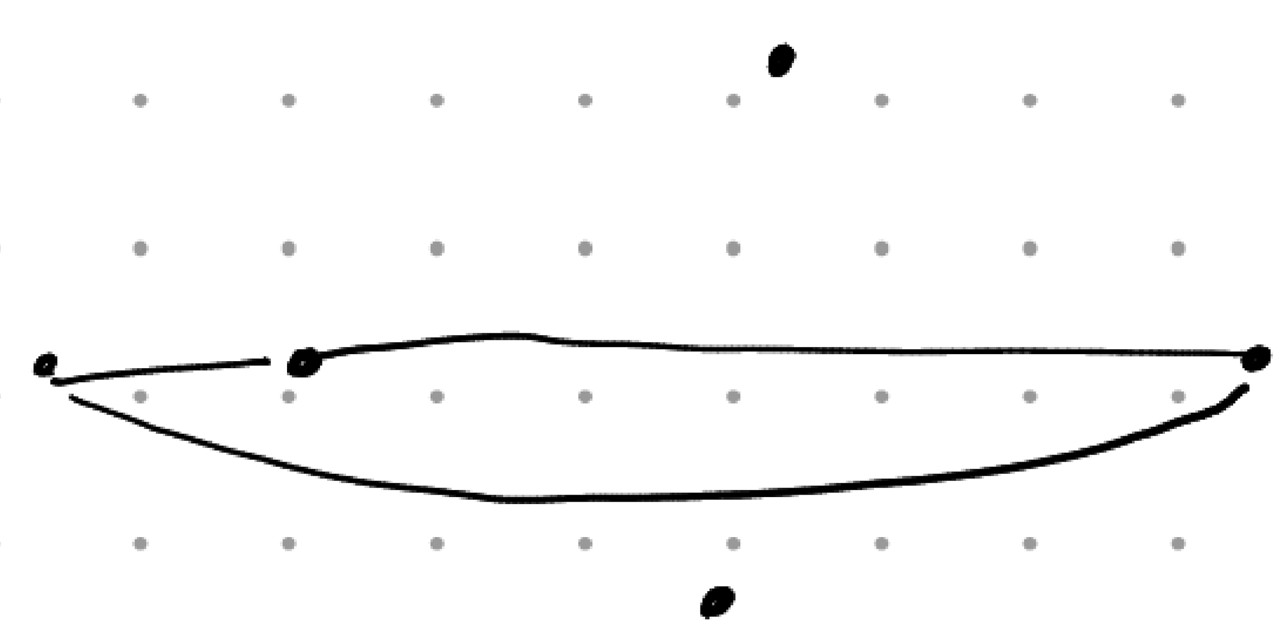
Mostafa Touny mtouny@msa.edu.eg
Salma Abdelhalim salmaemad@aucegypt.edu
Ali Nadeem nadim99@aucegypt.edu
Omar Shaalan shaalan@aucegypt.edu

Exercise 1

(i) order of G, $|G| = 5$. Size of G is 7
degree sequence $= (4, 4, 2, 2, 2)$
Complement of G : G'



$|G'| = 5$
Size of $G' = 2|G| - $ Size of G
$= 10 - 7 = 3$
deg. seq $= (2, 2, 2, 0, 0)$

(ii) Givens $|G| = n$, Size K
deg. seq. $= (K_1, \ldots, K_n)$
G' is the complement of G
$|G'| = |G| = n$
Size of G' is $2n - K$
deg. seq. $(n-1-K_n, n-1-K_{n-1}, \ldots, n-1-K_1)$

Exercise 2

True
Proof.
Since a graph is regular iff $\delta(G) = \Delta(G) = r$, Then
$\delta(G) = 2|E|/|V| = \Delta(G)$
$|V| = \frac{2|E|}{r} = \frac{2|E|}{11}$
$|V|$ is integer, so $|E|/11$ must be integer also. Call it m. Then $|V| = 2m$
That implies order of G, $|V|$, is even

Exercise 3

$(\rightarrow)$ $N[S] = N(S) \cup \{S\} = N(S)$

So $\{S\} \subset N(S)$, So for every vertex $v \in S$, both v and $N(v) \in N(S) = N[S]$

So v is a neighbour of another vertex $u \in S$.

Therefore, The graph generated by vertices in S must have at least one edge. Thus $\delta(\langle S \rangle) \geq 1$ (1)

$(\leftarrow)$ $\delta(\langle S \rangle) \geq 1$ means the min deg of the graph generated by vertices in S is at least one implying every vertex $u \in S$ is a neighbour for at least one other vertex $v \in S$

Thus $\{S\} \subset N(S)$, implying every vertex $\in N(S)$ must exist in $N[S]$, and every vertex in $N[S]$ must exist in $N[S]$. Hence $N(S) = N[S]$ (2)

Exercise 4

(i) Assumption. Graph G has at least a vertex

Fact. if $\deg v = K$, and we Know v has neighbours $v_1, v_2, \dots, v_m$, then v has at least another additional $K-m$ neighbour vertices $v_{m+1}, \dots, v_K$

Theorem. Main Problem

Select v_0 which exist by assumption. By hypothesis it has K neighbours. Select one of them as v_1. By the fact it has $K-1$ neighbours, none of which is v_0. Select v_2. if we continued, We will construct a path $(v_0, v_1, \dots, v_K)$ of length K

(ii) "failed to prove"

Exercise 5

let G be a closed odd walk, Then it's either a cycle Since first and last vertices are equal. Then we're done if already a cycle.

otherwise there are two interior equal vertices, Constructing two interior closed walks, and one of them must be an odd walk, as odd = odd + even.

by Considering the odd one, we re-check if it's already an odd cycle or not. if we continue the process, we end with an odd cycle