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Handin: Gradescope

Graph Theory
Homework Series 03
In Groups of 2-4 Students

Keywords: Connectivity, Bipartite Graphs, Isomorphism.

Exercise 1 (Isomorphisms, Bipartite Graphs)**.** *(1) Show that the isomorphism relation between graphs is an equivalence relation.*

(2) True or false: If $G \cong H$ and G is bipartite, then so is H. (Prove your answer)

3+2 points.

Exercise 2 (Connectivity)**.** *Prove that a graph G has no cut set if and only if G is a complete graph, i.e. $G = \mathcal{K}_n$ for some $n \in \mathbb{N}$.*

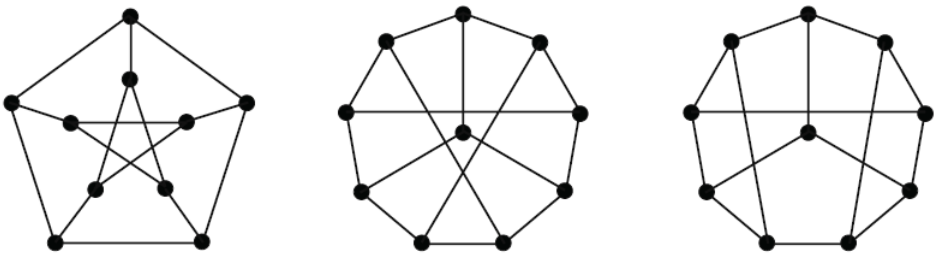
4 points.

Exercise 3 (Isomorphisms)**.** *Two of the graphs below are isomorphic.*

1) Which two are isomorphic?

2) For the two graphs which are isomorphic, give an appropriate isomorphism.

3) For the remaining graph, prove that it is not isomorphic to either of the other two.



Hint: It can be fun and useful to draw the graph with https://yworks.com/yed-live/ *and drag the vertices around to make them look the same.*

3+2+4 points.

Exercise 4 (Connectivity, ElKholy Conjecture)**.** *Decide whether the following is true or false and prove your answer:*

If G is a graph and $v \in V_G$ with $deg(v) = \delta(G)$, then v is not a cut vertex.

2 points.