Graph Theory
Homework Series 04
In Groups of 2-4 Students

Keywords: Distances, Adjacency Matrix, Trees.

Exercise 1 (Distance in Graphs)**.** *Let G be a connected graph. Show that (G, d) is a metric space.*

1+1+2 points.

Exercise 2 (Periphery, Eccentricity and Radius)**.** *Show the following three facts for an arbitrary graph G.*

(i) Assume $v \in P(G)$ and $d(u, v) = ecc(v)$. Then $u \in P(G)$.

(ii) If v and u are adjacent vertices, then their eccentricity differs by at most one.

(iii) Find a graph of order 7 which has radius 3 and diameter 4.

2+2+2 points.

Exercise 3 (Adjacency Matrix)**.** *Let G be any graph and A_G its adjacency matrix.*

(i) Prove that if $V_G = \{v_1, \dots, v_n\}$ then

$$d(v_i, v_j) := \min\{k \mid A_G^k(i, j) \neq 0\}.$$

(ii) Give a description of $C(G)$ and $P(G)$ in terms of the stroll matrix and argue your answer.

2+4 points.

Exercise 4 (Trees, Forests)**.** *Prove the following two claims.*

(i) Any forest of order n consisting of k many connected components has exactly $n-k$ many edges.

(ii) In any tree T, any edge is a bridge.

2+2 points.