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AUC Cairo
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Handin: Gradescope

Graph Theory
Homework Series 05
In Groups of 2-4 Students

Keywords: Properties of Trees, Spanning Trees, Minimum Weight Spanning Trees.

Exercise 1 (Trees, Spanning Trees).

(a) Assume G is a graph of order n with exactly $n-1$ many edges. Show that G is acyclic if and only if G is connected.

(b) Let G be any graph. Give a condition which is both necessary and sufficient for G to contain a spanning tree as an ***induced*** *subgraph. Prove your answer.*

2+2 points.

Exercise 2 (Trees and Leaves). *Let T be a tree of order at least 2.*

(a) Prove that if $\Delta(T) = k$, then T contains at least k many leaves.

(b) Prove that actually, there are exactly

$$2 + \Sigma_{v \in V(T^-)} (\deg(v) - 2)$$

many leaves in T, where $\deg(v)$ is the degree of v in T.

(c) Assume that every neighbor of a leaf in T has degree at least 3. Show that there are at least two leaves of distance 2 to each other.*

4+4+(2 Bonus) points.

Exercise 3 (Minimum Weight Spanning Trees). *Let (G, w) be a connected weighted graph in which distinct edges have distinct weights. Prove that there is a unique minimum-weight spanning tree.*

4 points.

Exercise 4 (Spanning Trees, Bridges). *Let G be connected and e and edge of G. Prove that e is a bridge if and only if e appears in any possible spanning tree of G.*

4 points.