Graph Theory
Homework Series 07
In Groups of 2-4 Students

Keywords: Hamiltonian Graphs, Independence Number, forbidden subgraphs.

Exercise 1. *Consider a graph G of order at least 4. Given the following conditions, is G Hamiltonian? Why?*

(a) *Assume G is complete bipartite graph $K_{n,n}$.*

(b) *Assume G be a 2-connected graph such that any four vertices contain a triangle (i.e. among any four vertices $\{v_1, v_2, v_3, v_4\}$ there are three which are mutually connected).*

(c) *Assume G is 2-connected and such that any three vertices, at least two are adjacent.*

9 points points.

Exercise 2. *Prove the following statements.*

(a) *Let $S \subseteq V(G)$ be nonempty. Then S is an independent set in G if and only if $\langle S \rangle^{\overline{G}}$ is a complete graph in the complement $\overline{G}$ of G.*

(b) *Any graph G is E_k-free if and only if $\alpha(G) < k$.*

6 points.

Exercise 3. *Prove or disprove the following claim: A graph G is claw-free if and only if $\Delta(G) \leq 2$.*

2 points.

Exercise 4. *Show that for any graph H there exists some graph G which is Hamiltonian and contains H as an induced subgraph. OR: Show that there exist a 2-connected, Hamiltonian graph which contains both $K_{1,3}$ and N as induced subgraphs.*

3 points.