Homework 07

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Contents

Exercises

1

(a). Yes. Let $v_1, \ldots, v_n, u_1, \ldots, u_n$ be the vertices. Then the cycle $(v_1, u_1, v_2, u_2, \ldots, v_n, u_n, v_1)$ is hamiltonian.

(b).

(c). No. A counter example is the graph whose vertices are v_1, v_2, v_3 and edges are $\{v_1, v_2\}$.

2

(a). If S is independent in G then for any $u, v \in S$, $\{u, v\} \notin E(G)$. By definition $\{u, v\} \in E(\overline{G})$. So $\{u, v\} \in \langle S \rangle^{\overline{G}}$. The converse holds by symmetry.

(b). If G is not $E_k - free$ then G has E_k as an induced subgraph, which in turn is an independent set. It follows $\alpha(G) \geq k$. If G is $E_k - free$ then by definition it does not contain an independent subgraph whose order is at least k. So the largest possible independent subgraph is strictly less than k. It follows $\alpha(G) < k$.

3

The statement is true. If a graph contains a claw as an induced subgraph, Then some vertex has three edges. In other words, Its degree is at least 3. So $\Delta(G) \not\leq 2$. If $\Delta(G) > 2$ then the graph has a vertex with at least 3 edges. The graph induced by that vertex alongside its 3 neighbour vertices is a claw. So G is not claw-free.

4

Denote the vertices of H by $u_1, u_2, \ldots, u_n$. Construct G, containing H as a subgraph, but with new vertices $\{v_i\}$ and new edges of the path $(v_1, u_1, v_2, u_2, \ldots, v_n, u_n, v_1)$.