Graph Theory
Homework Series 08
In Groups of 2-4 Students

Keywords: Planar Graphs, Eulers Formula, Kuratowski's Theorem.

Exercise 1 (Partite Graphs)**.** *Define the* ***multipartite graph*** $K_{r_1,r_2,\ldots,r_k} = (V,E)$ *where* V *is the disjoint union of the sets* $A_1,\ldots,A_k$ *such that* $|A_i| = r_i$ *and*

$$E = \{x_i x_j \mid x_i \in A_i, x_j \in A_j, i \neq j\},$$

i.e. E *contains all possible edges between vertices of different sets.*

(1) *Give a necessary and sufficient criterion on* r *and* s *such that* $K_{r,s}$ *is planar.*

(2) *Assume* $k > 2$. *Give a necessary and sifficient criterion on the* r_i *such that* $K_{r_1,r_2,\ldots,r_k}$ *is planar.*

3+3 points.

Exercise 2 (Subdivisions)**.** *Let* G *be a any graph and* $y \in V(G)$ *of degree* 2 *with* $N(y) = \{x,z\}$. *Let* $H := G - y + xz$ *be the graph arising from* H *by deleting* y *and all its incident edges and adding the edge* $e = xz$ *if necessary.*

(a) *Show that* G *is a subdivision of* H *if and only if* xz *is not an edge in* G.

(b) *Use (a) to show that any graph* G *is the subdivision of a graph* H *which itself is not a proper subdivision of any other graph.*

(c) *Describe all possible graphs* G *with* $\Delta(G) \leq 2$, *which are not a proper subdivision of any other graph.*

2+3+(2 Bonus) points.

Exercise 3 (Eulers Formula)**.** (a) *Let* G *be a planar,* 3*-regular graph of order* 24. *How many regions are in any planar representation of* G*?*

(b) *Let* G *be a connected planar graph of order less than* 12. *Prove that* $\delta(G) \leq 4$.

2+3 points.

Exercise 4. *Consider the graph G as given below. Is G planar? Prove your answer.*

4 points.