Graph Theory
Homework Series 09
In Groups of 2-4 Students

Keywords: Chromatic Number and Brooke's Theorem

Exercise 1 (Chromatic Number of Cycles)**.** *Prove that any cycle C_n is 3-colourable and C_n is 2-colourable if and only if n is even.*

4 points.

Exercise 2 (Chromatic Number and Independence)**.** *Let G be any graph.*

(a) Show that $\frac{|G|}{\alpha(G)} \leq \chi(G)$.

(b) Show that $\chi(G) \leq 1 + |G| - \alpha(G)$.

Hint for b: Pick an appropriate labelling of G and run the greedy algorithm.

4+4 points.

Exercise 3 (Brooke for disconnected graphs - Omar's Question)**.** *Give a necessary and sufficient condition for an arbitrary (i.e. possibly disconnected) graph to satisfy*

$$\chi(G) = \Delta(G) + 1.$$

Prove your answer.

4 points.

Exercise 4 (Applications of Colourings)**.** *Assume there are 7 committees with the members as below. How many meeting times do we need to schedule so that all committees can meet with all their members present?*

- $C_1 := \{Aschenbrenner, Borovic, Cherlin\}$
- $C_2 := \{Cherlin, Deloro, Evans\}$
- $C_3 := \{Deloro, Frecon\}$
- $C_4 := \{Aschenbrenner, Ghadernezhad\}$

- $C_5 := \{Evans, Hrushovski\}$
- $C_6 := \{Evans, Borovic, Ghadernezhad\}$
- $C_7 := \{Cherlin, Frecon, Hrushovski\}$.

4 points.

Exercise 5. *Let G be any graph of order n. Prove that*

(a) $n \leq \chi(G)\chi(\bar{G})$;

(b) $2\sqrt{n} \leq \chi(G) + \chi(\bar{G})$.

4 bonus points.