Graph Theory
Homework Series 11
In Groups of 2-4 Students

Keywords: Hall's Theorem, SDRs, König-Egerváry Theorem, Ramsey Theory.

Exercise 1 (König-Egerváry Theorem)**.** *Let G be a bipartite graph. Show that G has a matching of size at least $|E(G)|/\Delta(G)$.*

4 points.

Exercise 2 (SDRs)**.** *Let $\mathcal{F} := \{S_1, S_2, \ldots, S_r\}$ be a family of distinct nonempty subsets of $\{1, 2, \ldots, n\}$. Show that if all S_i are of the different cardinality, then there is a system of distinct representatives for $\mathcal{F}$.*

5 points.

Exercise 3 (König-Egerváry Theorem)**.** *Let $k \in \mathbb{Z}$ with $1 \leq k \leq n$. Further, let G be a subgraph of $K_{n,n}$ with more than $(k-1)n$ many edges. Prove that G has a matching of size at least k.*

4 points.

Exercise 4 (Ramsey Numbers)**.** *Prove that for any two positive integers $a, b \in \mathbb{Z}_+$ we have*

$$R(a, b) = R(b, a).$$

4 points.

Exercise 5 (Ramsey Numbers)**.** *Find $R(3, 3)$ and prove your answer.*

3 points.