Problem-Set 06

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The first part is solved by a trivial set-theoretic operations. The second is postponed.

Observe if $x \in f(\overline{E})$ then $x = f(p)$ for some $p \in \overline{E}$. To prove $x \in \overline{f(E)}$ it suffices to have $f(p) \notin f(E)$. By contrapositive, It follows by the given $p \notin E$.

Not the proof is independent of continuity.

Problem. 4

Observation. In case of a continuous function f at point p, If we have a sequence $\{x_i\}$ converging to p and $\{f(x_i)\}$ converges to a then $a = f(p)$.

Theorem. Our approach is proving the definition of a closed-set is satisfied. Namely, If p is a limit point of of $Z(f)$ then $p \in Z(f)$. Fix p and suppose $\forall \epsilon > 0\ \exists x \in Z(f)$ such that $d(x, p) < \epsilon$. We can construct a sequence x_i arbitrarily close to p where $f(x_i) = 0$. By the above observation it must be the case $f(p) = 0$, and hence $p \in Z(f)$.