Lab 06

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Contents

Exercises

6.1.1

(a).

Hints

- Sort the array as a preprocessing step.
- Given a sorted array, and an adjacent pair $A[i], A[i+1]$, Could the distance between $A[i]$ and $A[j]$ where $j > i+1$, be strictly less?
- Use that to design your algorithm.

Solution

```
# input: Array of integers
# output: minimum distance between any pairs
def ClosestDistance(A[0..n-1])

    # Transformation: Sort the array
    A.sort()

    # Initialize minimum distance to | A[0] - A[1] |
    minDistance = abs( A[0] - A[1] )

    # Iterate and compute the distance between adjacent elements
    for i in 1..n-1:
        currentDistance = | arr[i] - arr[i + 1] )

        # Update the minimum distance if the current distance is smaller
        if currentDistance < minDistance:
            minDistance = currentDistance

    # Return the minimum distance
    return minDistance
```

(b). **Homework**.

6.1.2

Homework.

6.2.4

We ask students whether $\Theta(n^3) - \Theta(n^3) + \Theta(n^3) = \Theta(n^3)$.

Hints

- Try to given a counter example where coefficients cancel each other.

Solution

We show it is not true in general true by the counter example $T_1(n) = n^3$, $T_2(n) = 2n^3$, and $T_3(n) = n^3$.

Analysis of the algorithm is left as a **homework**.

6.2.5

Homework.

6.3.5

(a)

Hints

- The idea is similar to binary search tree

Solution

```
# input: non-empty graph, by its root
# output: smallest element
def find_smallestKey(root):
    node = root

    while node.left is not None
        current = current.left

    return current.key

# input: non-empty graph, by its root
# output: largest element
def find_largestKey(root):
    node = root

    while node.right is not None:
        node = node.right

    return current.key

# input: non-empty graph, by its root
# output: difference between largest and smallest elements
```

```
def range(root)
    return find_largestKey(root) - find_smallestKey(root)
```

Complexity is $2 \log n = \Theta(\log n)$

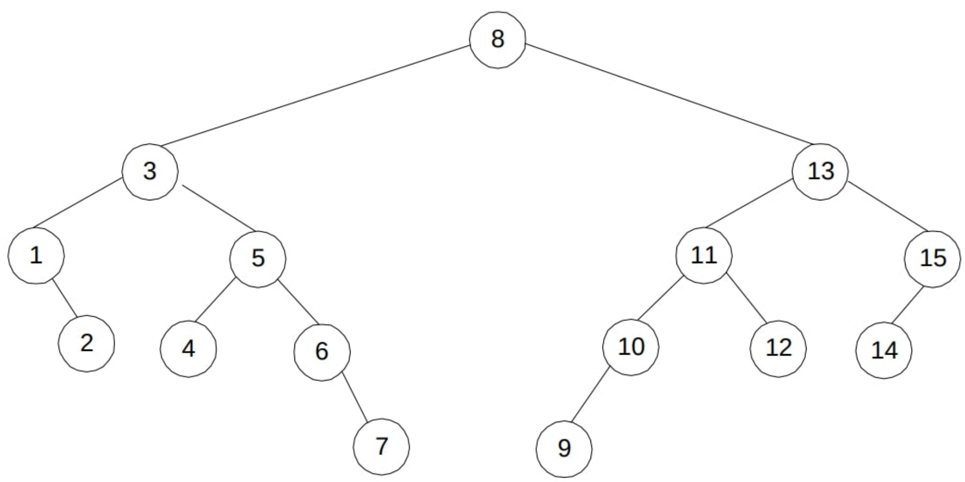
(b)

Hints

- For the largest, Note we can step down on left children. Similarly for the smallest, we can step down on right children.

Solution

False. Counter example from the solution manual.



6.3.9

Hints

- Very similar to binary search tree

Solution

Like the previous previous exercise we traverse left-most and right-most nodes. The difference is we consider left key and right key of these nodes, respectively.

```
def range(root)
    leftMost = find_leftMostNode(root)
    rightMost = find_rightMostNode(root)

    return rightMost.rightKey - leftMost.leftKey
```

6.4.2

Homework.

6.4.5

Students will be given the following subroutines.

```
# input: heap as an array, node by its index
# output: None. Given heap is modified in-place
def siftUp(heap, index):

    # cannot stif-up root node
    while index > 0:

        # parent of the node
        parentIndex = (index - 1) // 2

        # parental dominance is satisfied
        if heap[index] <= heap[parentIndex]
            break

        # if not satisfied, swap with parent
        swap(heap[index], heap[parentIndex])

        # set the cursor to the parent, and repeat
        index = parentIndex
```

```
# input: heap as an array, node by its index
# output: None. Given heap is modified in-place
def siftDown(heap, index):

    # Children indices
    leftChild_index = (2 * index) + 1
    rightChild_index = (2 * index) + 2


    # Find the largest out of index, leftChild_index, and rightChild_index

    # Initially set
    largest = index

    # Check if the left child exists. if larger, update largest
    if leftChild_index < len(heap) and heap[leftChild_index] > heap[largest]
```

```
        largest = leftChild_index

    # Check if the right child exists. if larger, update largest
    if rightChild_index < len(heap) and heap[rightChild_index] > heap[largest]:
        largest = rightChild_index


    # If the largest element is one of the children.
    if largest != index:

        # swap the child with parent
        swap( heap[index], heap[largest] )

        # recursively heapify the smaller tree
        siftDown(heap, largest)


    # parental dominance is satisfied here, whether recursion is called or not, so we
    return
```

(a). Homework.

Hint. Same logic of b but notably restrict search of the min element on leaves, $H[\lfloor n/2 \rfloor + 1], \ldots H[n]$. Also since the minimum is in leaves, we will only call `siftUp`.

```
  def delMin(heap H)
    # find the minimum node's index in leaves
    minElIndex = min(H[n/2 .. n])

    # swap the minimum with last node
    swapWithLast(minElIndex)

    # remove the last node
    removeLast()

    # sift-up the node in the index, previously containing the minimum
    siftUp(minElIndex )
```

(b).

Hints

- Use the element removal subroutine, given in the book. Call it `removeLast`.
- Use the swap with last indexed node trick, given in the book. Call it `swapWithLast`.

Solution

```
def findElementIndex(heap, target)
    for each element i of heap
        if i == target
            return i.index

def removeIndexNode(heap, index)

    # swap the indexed node with the last node
    swapWithLast(heap, index)

    # remove the last node
    removeLast(heap)

    # One of them must terminate in constant time
    siftDown(heap, index) # swapping downwards
    siftUp(heap, index) # swapping upwards

def removeElementNode(heap, target)

    # get the index of target by a linear scan
    index = findElementIndex(heap, target)

    # remove the element at found index
    removeIndexNode(heap, index)
```

It is easy to verify, that one of `siftDown` and `siftUp` must terminate in $\mathcal{O}(1)$, given the structure properties of the heap.

Complexity is $\mathcal{O}(n)+\mathcal{O}(1)+\mathcal{O}(1)+\mathcal{O}(\log n) = \mathcal{O}(n)$, respectively, of `findElementIndex` and `removeIndexNode`.

6.5.1

Homework.

6.5.9

We ask students how to compute the binary representation of a given number n.

```
def binaryRepresentation(n)

    # list storing binary representation
    # b[i] corresponds to ith digit
    binRep = []
```

```
    # by definition we know left-most digit is not 0
    # n becomes 0, only when last digit is computed
    while n != 0
      # fetch right-most digit
      b = n mod 2
      # eliminate right-most digit
      n = floor( n/2 )

      binRep.append(b)

    return binRep
```

Finally we hint to them, algorithm `RightToLeftBinaryExponentiation` in page 238 can be modified, so that it does not require list b(n) as an input.

6.6.5

Homework.

6.6.6

Homework.

6.6.+

You are given an array of positive integers. Find the maximum element but without using $>$ operator.

Hints

- Think of a related algorithm that uses $<$ operator
- Is the knowledge of minimum element useful in anyway?
- What if we transformed all elements to their negation?

Solution

```
def negationOfArray(A[0..n-1])
    for i in 0..n-1
        A[i] = -(A[i])

def minElement(A[0..n-1])
    minElement = A[0]

    for i in 1..n-1
```

```
        if A[i] < minElement
            minElement = A[i]

    return minElement

def maxElementByReduction(A[0..n-1])
    # transform
    negationOfArray(A)

    # conquer
    min = minElement(A)

    # solve the main problem
    return -(min)
```