Ch.12, Sec.1 - Rogawski & Adams' Calculus

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Contents

Ex. 1.66

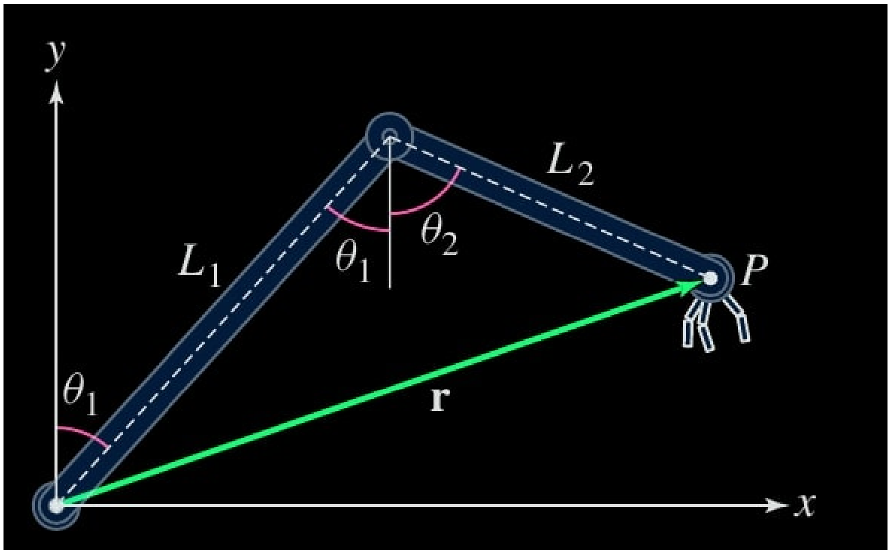
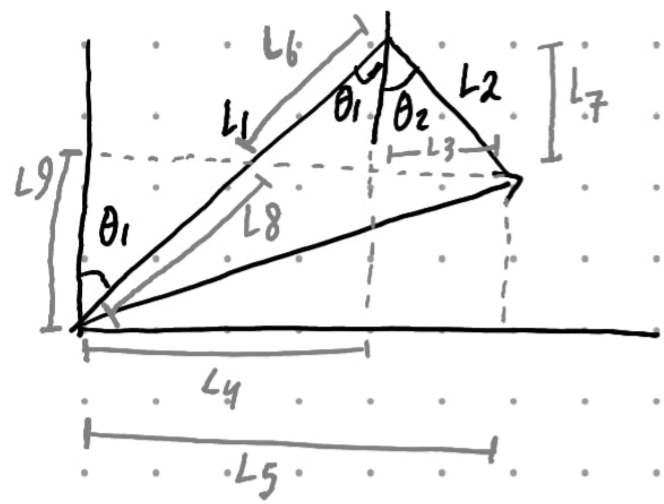


FIGURE 31

66. Find the components of the vector $\mathbf{r} = \overrightarrow{OP}$ in terms of θ_1 and θ_2.



$$||L_3|| = ||L_2|| \sin\theta_2$$
$$||L_4|| = ||L_6|| \sin\theta_1$$
$$||L_5|| = ||L_3|| + ||L_4||$$
$$||L_7|| = \sqrt{(L_2)^2 - (L_3)^2}$$
$$||L_7|| = ||L_6|| \cos\theta_1 \to ||L_6|| = ||L_7|| / \cos\theta_1$$
$$||L_8|| = ||L_1|| - ||L_6||$$
$$||L_9|| = ||L_8|| \cos\theta_1$$

Components are $\langle ||L_5||, ||L_9|| \rangle$

Ex. 1.69

69. Use vectors to prove that the diagonals $\overline{AC}$ and $\overline{BD}$ of a parallelogram bisect each other (Figure 32). *Hint:* Observe that the midpoint of $\overline{BD}$ is the terminal point of $\mathbf{w} + \frac{1}{2}(\mathbf{v} - \mathbf{w})$.

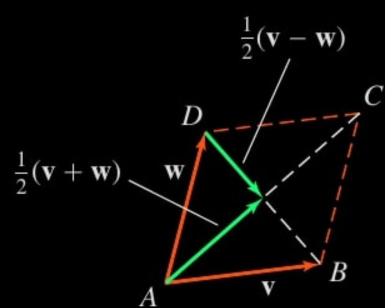


FIGURE 32

Observe $\overline{AC} = ||v + w||$ and $\overline{BD} = ||v - w||$.

By vector algebra,

$$w + \frac{1}{2}(v - w) = \frac{1}{2}(v + w)$$
$$\text{point of BD midpoint} = \text{point of AC midpoint}$$

Hence bisects each other.

Ex. 1.70

70. Use vectors to prove that the segments joining the midpoints of opposite sides of a quadrilateral bisect each other (Figure 33). *Hint:* Show that the midpoints of these segments are the terminal points of

$$\frac{1}{4}(2\mathbf{u}+\mathbf{v}+\mathbf{z}) \quad \text{and} \quad \frac{1}{4}(2\mathbf{v}+\mathbf{w}+\mathbf{u})$$

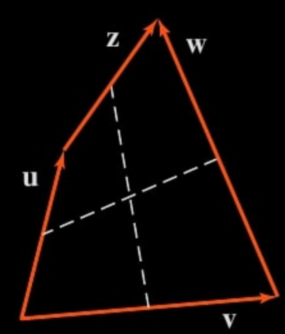
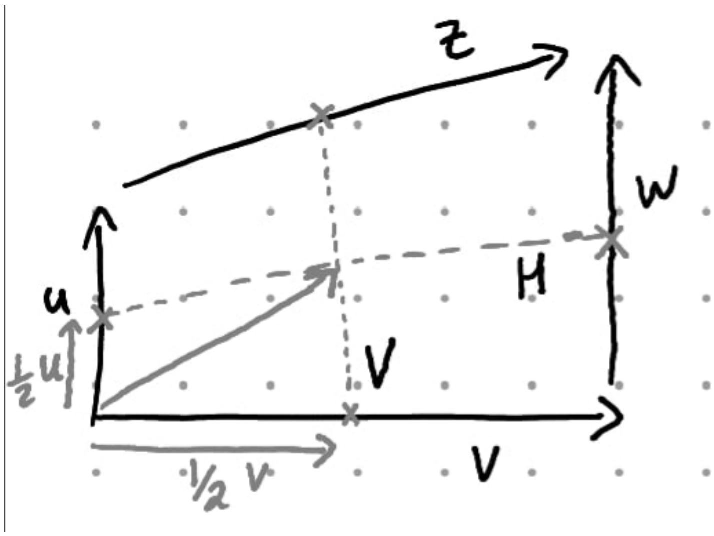


FIGURE 33



Observe that we can construct the midpoint between the ends of two vectors by $\frac{1}{2}(v-u)$.

The midpoint of H is $\frac{1}{2}[(v+\frac{1}{2}w)-\frac{1}{2}u]+\frac{1}{2}u=\frac{1}{2}v+\frac{1}{4}w+\frac{1}{4}u$.

The midpoint of V is $\frac{1}{2}[(u+\frac{1}{2}z)-\frac{1}{2}v]+\frac{1}{2}v=\frac{1}{2}u+\frac{1}{4}z+\frac{1}{4}v$

Suffices to show $2u+v+z=2v+w+u$ which reduces to $u+z=v+w$. By our diagram both yields exactly the same vector.

Note. I solved the problem without seeing the hint.

Ex. 1.71

71. Prove that two vectors $\mathbf{v} = \langle a, b \rangle$ and $\mathbf{w} = \langle c, d \rangle$ are perpendicular if and only if

$$ac + bd = 0$$

vectors v and w are perpendicular if and only if

$$\begin{aligned}
||w - v||^2 &= ||v||^2 + ||w||^2 \\
||\langle c - a, d - b \rangle||^2 &= a^2 + b^2 + c^2 + d^2 \\
(c - a)^2 + (d - b)^2 &= \\
c^2 + a^2 - 2ac + d^2 + b^2 - 2bd &= \\
-2ac - 2bd &= 0 \\
ac - bd &=
\end{aligned}$$