Chapter 01

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Contents

Problems

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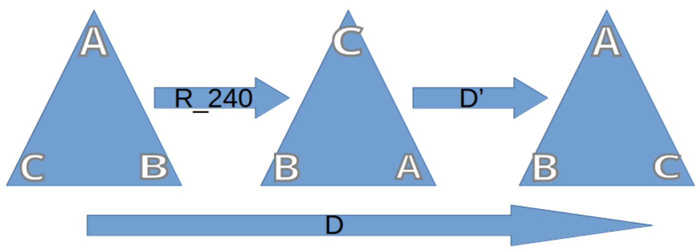
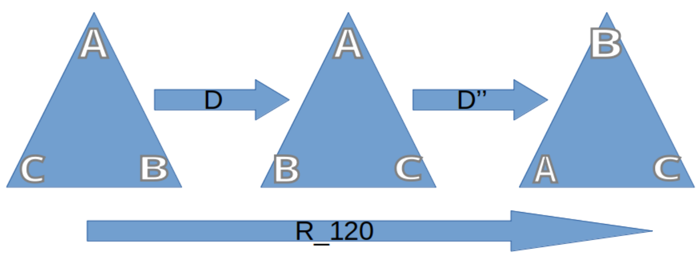
	R_0	R_{120}	R_{240}	D	D'	D''
R_0	R_0	R_{120}	R_{240}	D	D'	D''
R_{120}	R_{120}	R_{240}	R_0	D''	D	D'
R_{240}	R_{240}	R_0	R_{120}	D'	D''	D
D	D	D'	D''	R_0	R_{120}	R_{240}
D'	D'	D''	D	R_{240}	R_0	R_{120}
D''	D''	R_0	D'	R_{120}	R_{240}	R_0

Two pictures.





Not abelian.

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a. V.

b. R_{270}.

c. R_0.

d. R_0, R_{180}, H, V, D, D'.

e. None.

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We follow our intuition and generalize the cases of D_4 and D_3 with no formal argumentation.

For both cases, Elements include rotations $\frac{i}{n}360$ for $i = 1, 2, \ldots, n-1$. Counts n.

Even case only. Flips about the *ith* diagonal (counts $n/2$), and Flips about the *ith* axis (counts $n/2$)

Odd case only. Flips about the *ith* diagonal (counts n).

D_n is going to have a total of $2n$ elements; This fact was mentioned in the textbook though.

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Notation. We donate *Rotation* by T and *Reflection* by F.

Lemma. Through Caylay table in page 33, $TT = T$, $FF = T$, $TF = F$, and $FT = F$. In other words $X^2 = T$, and $XY = F$ if $X \neq Y$.

Theorem. Observe we can re-structure the given composed function as $a^2b^2b^2acc^2c^2a^2ac = TTTacTTTac = (TTTac)^2 = T$.

Therefore, Regardless of the choices of a, b, c, The given function is always a *rotation*.

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$D = HR_{90} = R_{90}V$.

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$X \neq H, V, D, D', R_0, R_{180}$, As otherwise $X^2 = R_0$ and then $Y = R_{90}$. For either of the remaining two cases $X = R_{90}$ or $X = R_{270}$, Necessarily $Y = R_{270}$.