$\frac{7.\cancel{7}5}{10}$

Chapter 05

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Problems

1

a

$$\alpha^{-1} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$$

b

$$\beta\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 2 & 3 & 4 & 5 \end{bmatrix}$$

c

$$\alpha\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 2 & 1 & 5 & 3 & 4 \end{bmatrix}$$

$\frac{1}{2}$

3(b)

b. (124)(35)(6)

$\frac{1}{2}$

6(a)

a. disjoint cycles form: (12)(356). Order is 6. ← lcm(2,3)

$\frac{1}{2}$

8(c,d)

~~is required 8 is not~~

even **c.** (17)(16)(15)(13).

odd **d.** (24)(23)(15).

$\frac{1}{2}$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 4 & 2 & 1 \end{bmatrix}$$

10

We want to find some permutation

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ & & & & & \end{bmatrix}$$

where:

- Order is 15, i.e lcm of disjoint cycles lengths is 15, and
- Even, i.e Has an even number of 2-cycles.

Observe $15 = 3 \cdot 5$ which suggests two disjoint cycles of lengths 3 and 5. A standard candidate is (123)(45678). Its 2-cycles form: (48)(47)(46)(45)(13)(12), A total of even 6 cycles.

2

The permutation in matrix form is:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 1 & 5 & 6 & 7 & 8 & 4 \end{bmatrix}$$

1

17

You need the subset to be non-empty

Using *theorem 3.3* (page 64), It suffices to show the set of even permutations are closed under permutation composition. By definition, Given even permutations $\alpha = (ab)(cd)\dots(ef)$ and $\beta = (gh)(ij)\dots(kl)$, The composition $\alpha\beta = (ab)\dots(kl)$ consists of even number of 2-cycles, As even + even = even.

0.75 / 1.5

23

The argument is incomplete.

Let H be a subgroup of S_n. Assume not every member is even. We show H must have an equal number of even and odd members.

1.5 / 3

We follow the same proof approach of *theorem 5.7* (page 104). We know there is an odd member α. For every odd β, $\alpha\beta$ is even, So there as many evens as there are odds. For every even β, $\alpha\beta$ is odd, So there are as many odds as there are evens. Therefore, the number of even and odd members are equal.

That is left multiplication is bijective in that case. For those to hold you need to say that if $\alpha\beta_1 = \alpha\beta_2$ then $\beta_1 = \beta_2$

25

The identity permutation is even. Not closed as the composition of two odd members is even.

$\frac{1}{2}$

43

For $n \geq 3$, It is easy to see $(12) \in S_n$ and $(23) \in S_n$. However

$$(23)(12) = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & \cdots \\ 3 & 1 & 2 & 4 & 5 & \cdots \end{bmatrix} \neq \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & \cdots \\ 2 & 3 & 1 & 4 & 5 & \cdots \end{bmatrix} = (12)(23)$$

$\frac{3}{2}$