Mathematical Logic
Assignment 1

Note:

You are expected to write proofs for the questions that asks you to compute or to find something.

Exercise 1

(i) Suppose that $\mathbf{P} = \{r\}$. Find the cardinality of $\mathcal{F}_n$.

(ii) Find a formula φ of maximum length where each symbol of φ belongs to the set $\{\, p, \wedge, \,), (\,\}$ and where the height of φ is 4.

(iv) Given two natural numbers m and n, what is the length of a propositional formula that has n occurrences of symbols for binary connectives and m occurrences of the symbol for negation?

Exercise 2

Consider formulas on a set of propositional variables $\mathbf{P}$. Given a natural number n, determine the possible different lengths of a formula whose height is n.

Exercise 3

Which of the following are well formed propositional formulas? Justfiy your answer.

1. $(\neg(pr \to (q = p)))$
2. $((p \wedge \neg q) \vee (q \to r))$

Exercise 4

Prove that the following hold For all words w, w_1, w_2, w_3, w_4, on an arbitrary, non-empty alphabet.

- If $ww_1 = ww_2$, then $w_1 = w_2$.
- If $w_1w_2 = w_3w_4$, then either w_1 is an initial segment of w_3 or else w_3 is an initial segment of w_1.
- $\ell(w_1w_2) = \ell(w_1) + \ell(w_2)$.