Note:

You are expected to write proofs for the exercises that ask you to compute or to find something.

Exercise 1 (5+5+5)

Use the decomposition trees to find all subformulas of each of the following formulas :

1. $\neg(\neg p \leftrightarrow r)$
2. $((p \wedge r) \rightarrow (\neg p \leftrightarrow q))$
3. $\neg((\neg r \vee (r \wedge \neg p)) \leftrightarrow \neg\neg\neg q)$

Exercise 2 (10)

Show that number of occurrences of the symbol $\wedge$ in a formula φ is less than or equal to the number of closing parenthesis.

Exercise 3 (5+5+5+5)

1. Let δ be the truth assignment defined on the set of propositional variables $\{p, q, r\}$ by $\delta(p) = 1$, $\delta(q) = 0$, $\delta(r) = 0$. Find the truth value under δ of each of the following formulas:
   (a) $\psi(q) = \neg q$
   (b) $\theta(p, r) = (\neg p \vee r)$
   (c) $\gamma(p, r) = (p \leftrightarrow (\neg r \rightarrow p))$
2. Find the formula $\gamma(\gamma/p, \theta/r)$

Exercise 4 (20)

Show that for any formula φ, we have that $h[\neg\varphi] = h[\varphi] + 1$.

Exercise 5 (15)

Show that for any $\varphi, \psi \in \mathcal{F}$: $\varphi \equiv \psi$ if and only if $(\phi \leftrightarrow \psi)$ is a tautology.

Exercise 6 (10+10)

1. For any truth assignments δ and λ and any formula $\varphi(p_1, p_2, ..., p_n)$, if δ and λ agree on the set $\{p_1, p_2, ..., p_n\}$, prove that $\delta[\varphi] = \lambda[\varphi]$.
2. Suppose $|P| = n$ for some positive integer n. Compute $| \mathcal{F}/_{\equiv} |$ (i.e How many $\equiv$-equivalence classes on $\mathcal{F}$ are there?).