Mathematical Logic
Assignment 3

Note:

You are expected to write proofs for the exercises that ask you to compute or to find something.

Exercise 1 (20)

Determine which of the following logical equivalences hold:

1. $\neg(p \wedge r) \equiv (\neg p \vee \neg r)$
2. $((p \wedge q) \wedge r) \equiv (p \wedge (q \wedge r))$
3. $((p \vee q) \vee r) \equiv (p \vee (q \vee r))$
4. $((p \rightarrow q) \rightarrow r) \equiv (p \rightarrow (q \rightarrow r))$

Exercise 2 (30)

Determine for each of the following formulas whether they are tautologies, contradictions or neither. Then find, for each, an equivalent formula in DNF.

1. $(p \vee \neg p)$
2. $(((p \rightarrow q) \wedge \neg q) \rightarrow \neg p)$
3. $((\neg p \rightarrow p) \leftrightarrow p)$
4. $(\neg p \rightarrow (\neg q \leftrightarrow (q \rightarrow p)))$

Exercise 3 (25)

For a formula ϕ built up using the connectives $\{\neg, \wedge, \vee\}$, let ϕ^* be constructed by replacing any literal by its negation (i.e replacing it with a negation symbol concatenated by the same literal).

1. For any truth assignment δ, let δ^* be the truth assignment defined for any propositional variable p as follows:
$$\delta^*(p) = \begin{cases} 1 & if \delta(p) = 0 \\ 0 & if \delta(p) = 1 \end{cases}$$
Show that $\delta(\phi) = \delta^*(\phi^*)$. [Hint: This is a statement about all formulas of a certain sort]
2. Show that ϕ is a tautology if and only if ϕ^* is a tautology.