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MACT 4133

Mathematical Logic
Assignment 5

AUC

Note:

You are expected to write proofs for the exercises that ask you to compute or to find something.

Exercise 1 (8+8)

Decide which of the following are derivable in S:

1. $\neg(q \to \neg p) \vdash ((\neg q \to p) \to \neg(q \to \neg p))$
2. $\vdash ((p \to (p \to \neg r)) \to (q \to r))$

Exercise 2 (8+8+8)

Show which of the following are consistent and which are not:

1. $\{\neg(p \to q), \neg(q \to r)\}$
2. $\{(p \to q), (q \to r), (r \to \neg p)\}$
3. $\{\neg(p \to q), q\}$

Exercise 3 (20)

Let Γ be a set of formulas and put $\Sigma = \{\varphi \mid \Gamma \vdash \varphi\}$. Show that $\Gamma \vdash \varphi$ if and only if $\Sigma \vdash \varphi$.

Exercise 4 (15)

Let $\delta : \mathcal{F}^* \to \{0, 1\}$ be a truth assignment and put $\Sigma_\delta = \{\varphi \in \mathcal{F}^* \mid \delta[\varphi] = 1\}$. Show that Σ_δ is complete.

Exercise 5 (7+13+20+20+20+20)

Definition: Let Γ be a set of propositional formulas. We say that Γ is independent if for every formula $\phi \in \Gamma$ we have $\Gamma \backslash \{\phi\} \not\models \phi$.

1. Is the empty set independent?
2. Which singletons are independent and which are not? Provide necessary and sufficient condition for a singleton to be independent (i.e. if and only if condition).
3. We define $\Gamma, \Delta \subseteq \mathcal{F}$ to be equivalent when for each truth assignment δ, δ satisfies Γ if and only if it satisfies Δ. Show that every finite set of formulas has at least one independent equivalent subset.
4. Show that a set of formulas is independent if and only if every finite subset of it is independent.
5. Are maximally consistent sets independent?
6. Provide necessary and sufficient condition for inconsistent sets to be independent.