Mathematical Logic
Assignment 8

Note:

You are expected to write proofs for the exercises that ask you to compute or to find something.

Exercise 1 **(10+10)**

Find derivations for each of the following in $\mathcal{S}_{\mathcal{L}}$:

1. $\vdash (x = y \to (y = z \to x = z))$
2. $\vdash (\forall x\, \forall y\, \varphi \to \forall y\, \forall x\, \varphi)$

Exercise 2 **(20)**

Show that axiom 5 of the system $\mathcal{S}_{\mathcal{L}}$ is logically valid.

Exercise 3 **(25)**

Let Σ be a set sentences which are only satisfiable by finite models. Use **only** the compactness theorem to show that there is $n \in \mathbb{Z}^+$ such that for any model $\mathcal{M}$ of Σ, the cardinality of M is at most n.

Exercise 4 **(15+20)**

1. Let $x_1, x_2 \ldots x_n$ be sets. Show that $\bigcup_{i=1}^{n} x_i$ is also a set.
2. Let x and y be sets. Show that the Cartesian product $x \times y$ is also a set, where (x, y) is $\{\{x\}, \{x, y\}\}$.